

9. i) Literal :- A literal is a variable available either in normal or complement form.

e.g. literals in Boolean expression:

$$A'B(0 + CD') + B(A + A'CD)$$

Sol. No. of literals = 10.

ii) fundamental Product (minterm) :- The product of all literals either with complement or without complement.

e.g. 1. xz' , $x'y_0z$, $xy'z$ are fundamental products

2. $xyx'z$ and $xyzzy$ are not fundamental products.

3. Reduce to fundamental product: i) $xyx'z$

$$\Rightarrow (xx')yz \quad (\because a \cdot a' = 0)$$

$$\Rightarrow (0)yz \Rightarrow 0$$

$$\text{ii) } xyzzy = xz(yy) \Rightarrow xz(y) \Rightarrow xyz.$$

$$\begin{aligned} \text{iii) } xy'zx'y' &= xzx(y'y') \\ &= (xx)z(y') \quad (\because a \cdot a = a) \\ &\Rightarrow xy'z \end{aligned}$$

iii) Maxterm :- maxterm of n variable is a sum of n variables in which each variable appears exactly once in either true or complemented form but not both

e.g. $x+y$, $x'+y$, $x+y'$, $x'+y'$, etc.
 $x'+y+z'$, $x'+y'+z$

iv) Sum of Product (disjunctive normal form or DNF) :-
(SOP)

A Boolean Expression E is called SOP if E is a fundamental product or the sum of two or more fundamental products none of which is contained in another.

e.g 1. $E(x, y, z) = xz' + y'z + xyz'$
it is not SOP $\because xz'$ contains xyz'

$$\begin{aligned} E(x, y, z) &= (xz' + xyz') + y'z \\ &= xz'(1 + y) + y'z \\ &= xz'(1) + y'z \\ &= xz' + y'z \rightarrow \text{which is S.O.P} \end{aligned}$$

or

*A Boolean Expression over two value Boolean algebra is said disjunctive normal form (SOP) if it is join of minterms.

e.g 2. $f(x, y, z) = (x' \wedge z) \vee (y \wedge z) \vee (y \wedge z')$
simplify and write in Minterm Normal form.

Sol.

$$\begin{aligned} f(x, y, z) &= (x' \wedge z) \vee [y \wedge (z \vee z')] \\ &= (x' \wedge z) \vee (y \wedge 1) \\ &= (x' \wedge z) \vee y \end{aligned}$$

x	y	z	x'	x' ∧ z	(x' ∧ z) ∨ y	minterm
0	0	0	1	0	0	m ₁
0	0	1	1	1	1	m ₂
0	1	0	1	0	1	m ₃
0	1	1	1	1	1	m ₄
1	0	0	0	0	0	m ₅
1	0	1	0	0	0	m ₆
1	1	0	0	0	1	m ₇
1	1	1	0	0	1	m ₈

minterms are $m_2 = x' \wedge y' \wedge z$

$$m_3 = x' \wedge y \wedge z'$$

$$m_4 = x' \wedge y \wedge z$$

$$m_7 = x \wedge y \wedge z', \quad m_8 = x \wedge y \wedge z$$

Hence minterm Normal form = $m_2 \vee m_3 \vee m_4 \vee m_7 \vee m_8$

$$\Rightarrow (x' \wedge y' \wedge z) \vee (x' \wedge y \wedge z') \vee (x' \wedge y \wedge z) \vee (x \wedge y \wedge z') \vee (x \wedge y \wedge z)$$

★. Complete Sum of Product :- A Boolean Expression

E is said to be Complete S.O.P if E is a sum of Product Expression where each Product involves all the n-variables.

A fundamental Product which involves all the variables is called minterm.

Q.3 Express E as SOP and Complete SOP.

i) $E = x(xy' + x'y + y'z)$

ii) $E = z(x' + y) + y'$

i) $E = x(xy' + x'y + y'z)$

$$= xx'y' + xx'y + xy'z$$

$$= xy' + 0 \cdot y + xy'z \quad [a \cdot a' = 0, a \cdot a = a]$$

$$= xy' + xy'z$$

$$= xy' \quad [\because xy' \text{ contain in } xy'z]$$

S.O.P

ii) $E = xy'(z' + z) \Rightarrow xy'z' + xy'z$
which is complete S.O.P.

ii) $E = z(x' + y) + y'$

$$= zx' + zy + y' \quad \text{which is S.O.P.}$$

$$E = x'z(y + y') + yz(x + x') + y'(x + x')(z + z')$$
$$= x'zy + x'zy' + yzx + yzx' + y'[xz + xz' + x'z + x'z']$$

$$= x'zy + x'zy' + yzx + yzx' + y'xz + y'xz'$$
$$+ y'x'z + y'x'z'$$

$$= x'yz + x'y'z + x'yz + x'y'z + x'yz + x'y'z'$$
$$+ x'y'z + x'y'z'$$

$$= xyz + (x'yz + x'yz) + (x'y'z + x'y'z) + x'y'z'$$
$$+ x'yz + x'y'z'$$

$$E = xyz + x'yz + x'y'z + x^2yz' + xy'z + x'y'z'$$

which is Complete S.O.P.

Q.4 Express $E = (x' + y)' + x'y$ in Complete S.O.P.

Sol.

$$\begin{aligned}
 E &= (x' + y)' + x'y \\
 &= (x')' y' + x'y \quad ((a+b)' = a'b') \\
 &= xy' + x'y \\
 &= xy'(z + z') + x'y(z + z') \\
 &= xy'z + xy'z' + x'yz + x'yz'
 \end{aligned}$$

which is Complete S.O.P of E .

Q.5 Express $E = x'y'z + xyz + y + yz' + x'z$ in Complete S.O.P

Sol.

$$\begin{aligned}
 E &= x'y'z + xyz + y(x + x')(z + z') + yz'(x + x') \\
 &\quad + x'z(y + y') \\
 &= x'y'z + xyz + y[xz + xz' + x'z + x'z'] + yz'x + yz'x' \\
 &\quad + x'zy + x'zy' \\
 &= (x'y'z + x'y'z') + (xyz + x'yz) + (yxz' + yxz') \\
 &\quad + (yx'z + x'zy) + (yx'z' + yx'z') \\
 &= x'y'z + xyz + x'yz' + x'yz + x'y'z'
 \end{aligned}$$

which is Complete S.O.P.